Classes 5. WolframAlpha: Linear algebra. Symbolic and numerical calculations

Solution of linear equations. Let's use the instruction *solve,*
for example,
solve $x+y-z=1, x-2y-z=-2, 3x-y+z=3$ or directly
$x+y=5,\ x-y=-1.$

Exercise 1. Solve systems of equations

a) $5x-7y+z=12,\ 2x+3y-4z=-1,\ 9x-y-5z=10;$
b) $5x-7y+z=12,\ 10x-14y+2z=1,\ x+y-5z=4.$

Matrix operations
Exercise 2. Calculate

a) rank matrix $\begin{bmatrix} 4 & 0 & 2 \\ 3 & 5 & 1 \\ 2 & -1 & 6 \end{bmatrix}$;

b) The inverse matrix for $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 2 & -1 & 0 \end{bmatrix}$.

Exercise 3. Calculate the determinant of the matrix

$$\begin{bmatrix} 1 & 3 & 0 & -1 \\ 2 & 4 & -2 & 4 \\ 3 & 1 & -1 & 7 \\ -1 & 3 & 8 & 3 \end{bmatrix}.$$

Exercises 4. Calculate the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 5 & 4 \\ 1 & -3 \end{bmatrix}.$$

Limits. In order to find the limit we use instruction *limit*, for example,
limit [f(n), n -> infinity].
Exercises 5. Find the limit:
a) $\lim_{n\to\infty} \frac{n^2}{n^3+5n}$;
b) $\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^n$.

For task 6 we use the limit instruction, when x tends to a:
Limit [f [x], x ->a].
Exercises 6. Find the limit:
a) $\lim_{x\to 0} \frac{2\sin x}{x}$;
b) $\lim_{x\to -1} \frac{1-x^3}{1+x^3}$.

Derivative

In order to find the derivative of the function $f(x)$ we use the instruction: D[f [x] x] .
To find a derivative of the n-th order, we should write D[f [x], {x, n}].
To find a partial derivative of a function $f(x,y,...,z)$ we write D[f [x, y,..., z], i], where i is a suitable variable.
If we want to find a partial derivative with respect to order n, then write D[f [x, y, ...,z], {i, n}], which means the same as above.

Exercises 7. Find the derivative:
a) $f'(x)$, where $f(x) = e^x \ln x$;

b) $f^{(v)}(x)$, where $f(x) = x^2\, e^x$;

c) $f'_x(x, y)$, where $f(x, y) = x^2\, y^3 \cos(x - y)$;

d) $f'_y(x, y)$, where $f(x, y) = x^2\, y^3 \cos(x - y)$;

e) $f'''(x)$, where $f(x) = \frac{2x}{x^2-4}$.

Integral

To find the indefinite integral of the function $f(x)$ you need to write: Integrate[f [x], x].
Finding the definite integral is very straightforward: Integrate[f [x], {x, a, b}].

Exercises 8. Find the indefinite integral:

a) $\int \frac{\cos x}{x^3}\, dx$;

b) $\int x^2 \arccos x\, dx$;

c) $\int_{-\pi/4}^{\pi/4} \frac{x+\mathrm{tg}\, x}{x^2}\, dx$;

d) $\int_2^{\infty} \frac{\ln(x^2+2)}{x^4}\, dx$.

Differential equations and their systems

To find the general solution of the differential equation $F\left(x, y, y', \ ..., y^{(n)}\right) = 0$ we use the instruction F [x, y, y', y', ...].
If we need to solve Cauchy's problem, we write
F [x, y, y', y', ...], y [$x0$] == $y0$, y'[$x0$] == $y1$, ...
If we want to solve the boundary problem, then we write the boundary conditions separated by commas in the form y[$x0$] == $y0$.
The solution of the system of differential equations is simple, just write:
{f_1, f_2, ..., f_n} where f_1, f_2, ..., f_n are differential equations occurring in the system.

Exercises 9. Find a general or particular solution of differential equation:

a) $y''' - y'' + y' - y = \cos x$;

b) $y'' - 2y' + y = \ln x$;

c) $y\, y'' - 2\,(y')^2 - y^2 = 0$;

d) $y'' + y = 0,\ y(0) = 1,\ y'(0) = 0$.

e) $y' + x\, y = x^2,\ y(1) = 1$;

f) $y''' - 3\, y'' + 3\, y' - y = 4\, x,\ y(0) = 0,\ y'(0) = 1, y''(0) = 0$;

g) $\{y' + z' = 8,\ y' - 4\, z' = 2\}$.

Exercises 10. Find the numerical solution of the equations:

a) $e^{x^2} = 3 \sin x$;

b) $\mathrm{tg}\, x = x^3$ (with an accuracy of 25 significant digits).